The Problem of Coordination

A Congestion Game Study

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Outline

- Game Theory
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- Analytic Results
- Discussion

Game Theory

- Game theory is a branch of applied mathematics that is used in the social sciences (most notably economics), biology, engineering, political science, international relations, computer science, and philosophy. Game theory attempts to mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others.
- Traditional applications of game theory attempt to find equilibria in these games. In an equilibrium, each player of the game has adopted a strategy that they are unlikely to change (e.g. Nash Equilibrium).
- Today, "game theory is a sort of umbrella or 'unified field' theory for the rational side of social science, where 'social' is interpreted broadly, to include human as well as non-human players (computers, animals, plants)" (Aumann 1987).

1. **Game Theory – wikipedia [http://en.wikipedia.org/wiki/Game_theory]**
2. **D. Ross "Game Theory". The Stanford Encyclopedia of Philosophy (Spring 2008 Edition)**
3. **B. Skyrms (1990). The Dynamics of Rational Deliberation, Harvard University Press.**
4. **"Knowledge, Belief, and Counterfactual Reasoning in Games." (1999). In Cristina Bicchieri, Richard Jeffrey, and Brian Skyrms, eds., The Logic of Strategy. New York: Oxford University Press.**

The Metaphor: the "Congestion Game"

1. B. Vocking. Congestion Games: Optimization in Competition [www-i1.informatik.rwth-aachen.de/~voecking/publications/ACiD06.pdf]

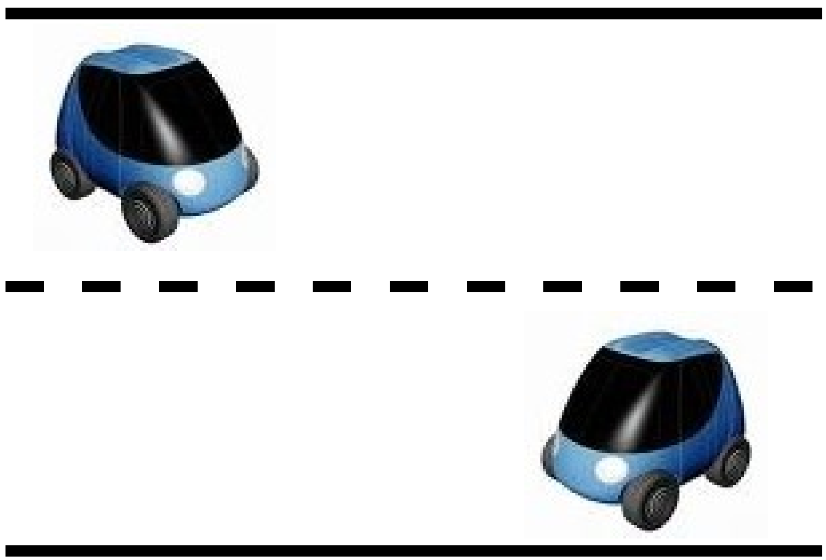
2. I. Milchtaich (1996). Congestion Games with Player-Specific Payoff Functions. *Games and Economic Behavior Vol. 13, Issue 1, Pages 111-124*

3. Congestion Games and Coordination Mechanisms (2004). Lecture Notes in Computer Science, Mathematical Foundations of Computer Science 2004 - ISBN 978-3-540-22823-3

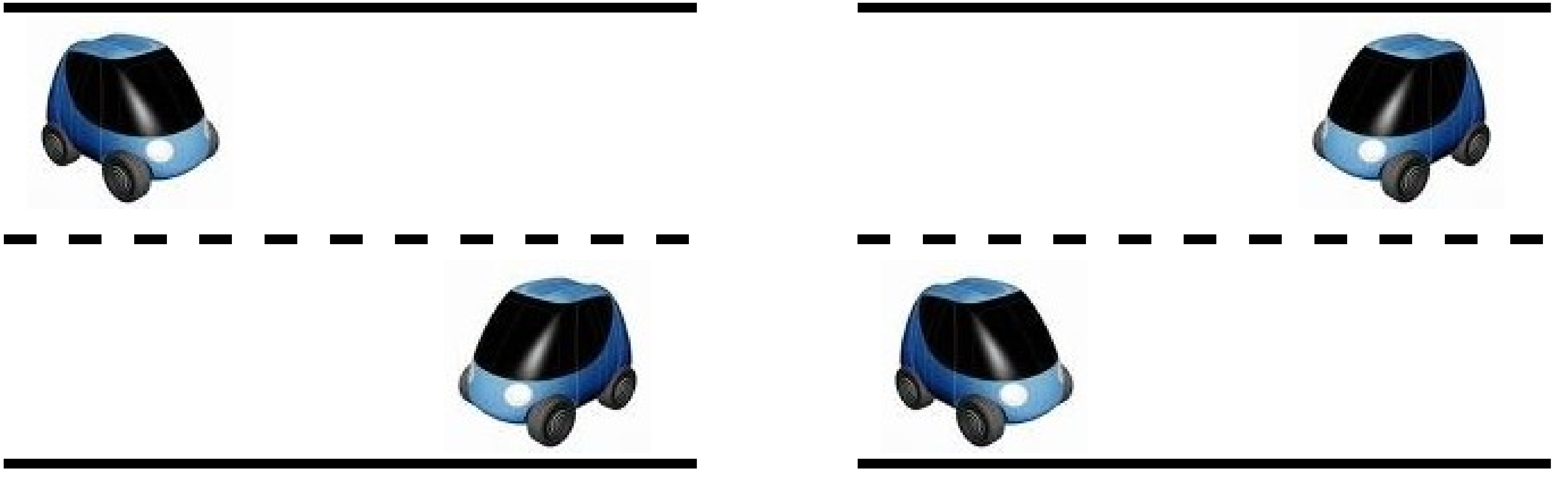
The World

Social Conventions

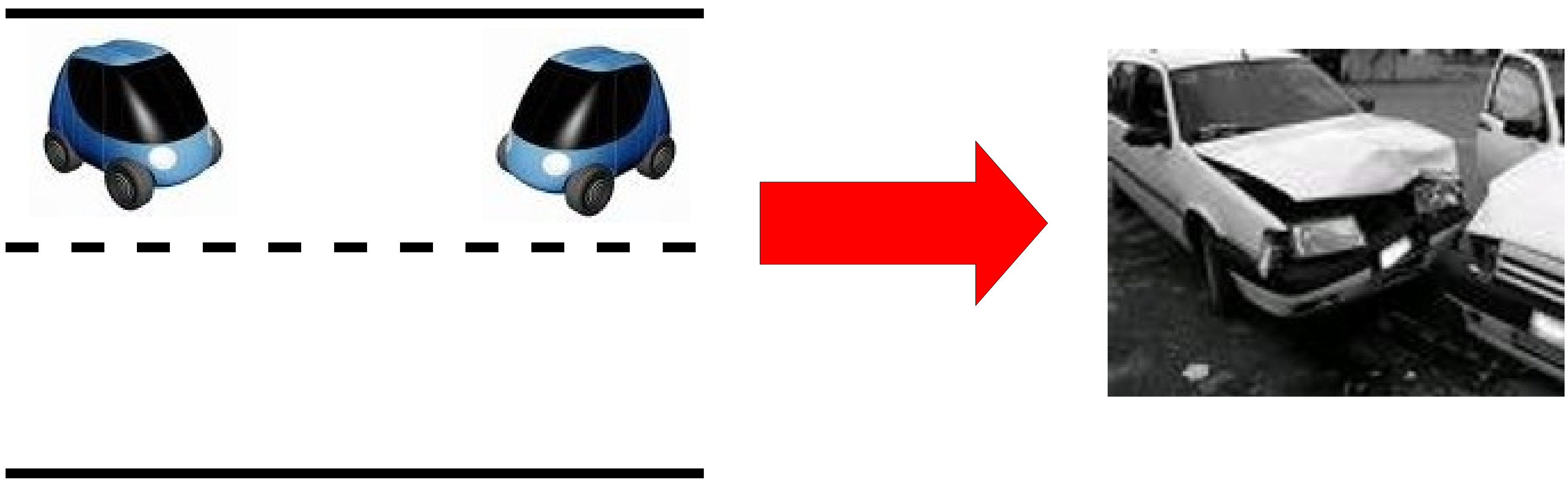
- A Social Convention is a *behavioral regularity*
- Conventions are a class of problems classified as *coordination games* (keep the right or left when driving), based on interdependency and mutual expectations



- A Convention coordinates people's expectations in socio-economic interactions that have multiple *equilibria*



- A Convention is a *pareto-optimal rational* choice



References – Social Conventions

1. **Y. Shoham and M. Tennenholtz (1997). On the emergence of social conventions: modeling, analysis, and simulations. *Artificial Intelligence*, Vol. 94, Pages: 139—166.**
2. **S. Sen and S. Airiau (2007). Emergence of norms through social learning. In Proceedings of the Twentieth International Joint Conference on Artificial Intelligence, pages 1507–1512.**
3. **D. K. Lewis. (1969). Convention: A Philosophical Study. Harvard University Press, Cambridge, MA.**
4. **J. Maynard Smith (1974) The theory of games and the evolution of animal conflicts. Journal of Theoretical Biology, 47:209–21.**
5. **J. Maynard Smith and G. R. Price. (1973). The logic of animal conflict. Nature, 246:15–18.**

Focus

- in coordination problem situations, rational agents may also converge on non-conventional equilibria
- compare simulative and analytical results (drawing some conclusions)

The Simulation's Algorithms (1)

Algorithm 1: The scheduling of simulation.

Let be:
- **a** is an agent
- **P** a society consisting of a single population with
- **N** individuals moving (random) over a discrete bi-dimensional lattice
- **L** with
- **c** cells for time **T,**
- with **Crossroads** = the number of cells with more than 1 individual

```
for t = 1 to T do
    for a = 1 to N do
      a ⇐ P.
      to set a over L
    end for
    for c = 1 to Crossroads do
      Compute payoffs
    end for
    for a = 1 to N do
      Imitation dynamic
    end for
end for
```

The Simulation's Algorithms (2)

Algorithm 2: Conditionated.

A is the individual. **X** indicate the direction to monitor. CRASH indicate that **A** goes into the crossroad at the same time of an other orthogonal individual. STOP indicate that **A** does not go into the crossroad. GOAHEAD indicate that the individual crosses the crossroad without that other orthogonal individual could make the same behavior. **H** is an Hawk individual. **D** is a Dove individual. We define not blocked a conditionated individual with the free monitored direction.

```
if X is occupied then
   STOP
else
   if there is (orthogonal H) or not blocked c then
      CRASH
   else
      GOAHEAD
   end if
end if
```

The Simulation's Algorithms (3)

Algorithm 3: Aggressive.

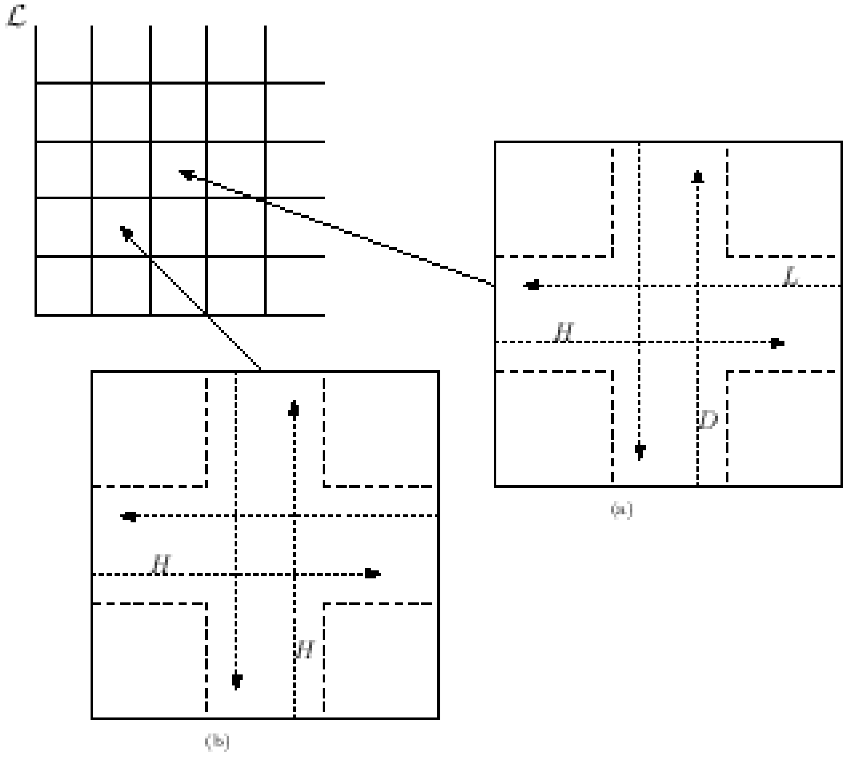
```
GOAHEAD
    if there is (orthogonal H) or not blocked C then
        CRASH
    end if
```

Algorithm 4: Compliant.

```
if there is orthogonal individuals then
  STOP
else
  GOAHEAD
end if
```

Some Constraints

- Individuals that are alone in a cell does not get payoff
- We have a maximum of four individual within a single cell
- We have only an individual for each direction (it is not possible that, for example, two Hawk's are running in North-South direction)
- Only the orthogonal individuals (we mean, individuals coming from orthogonal directions) matter, and then they could modify the payoff between them



Payoff

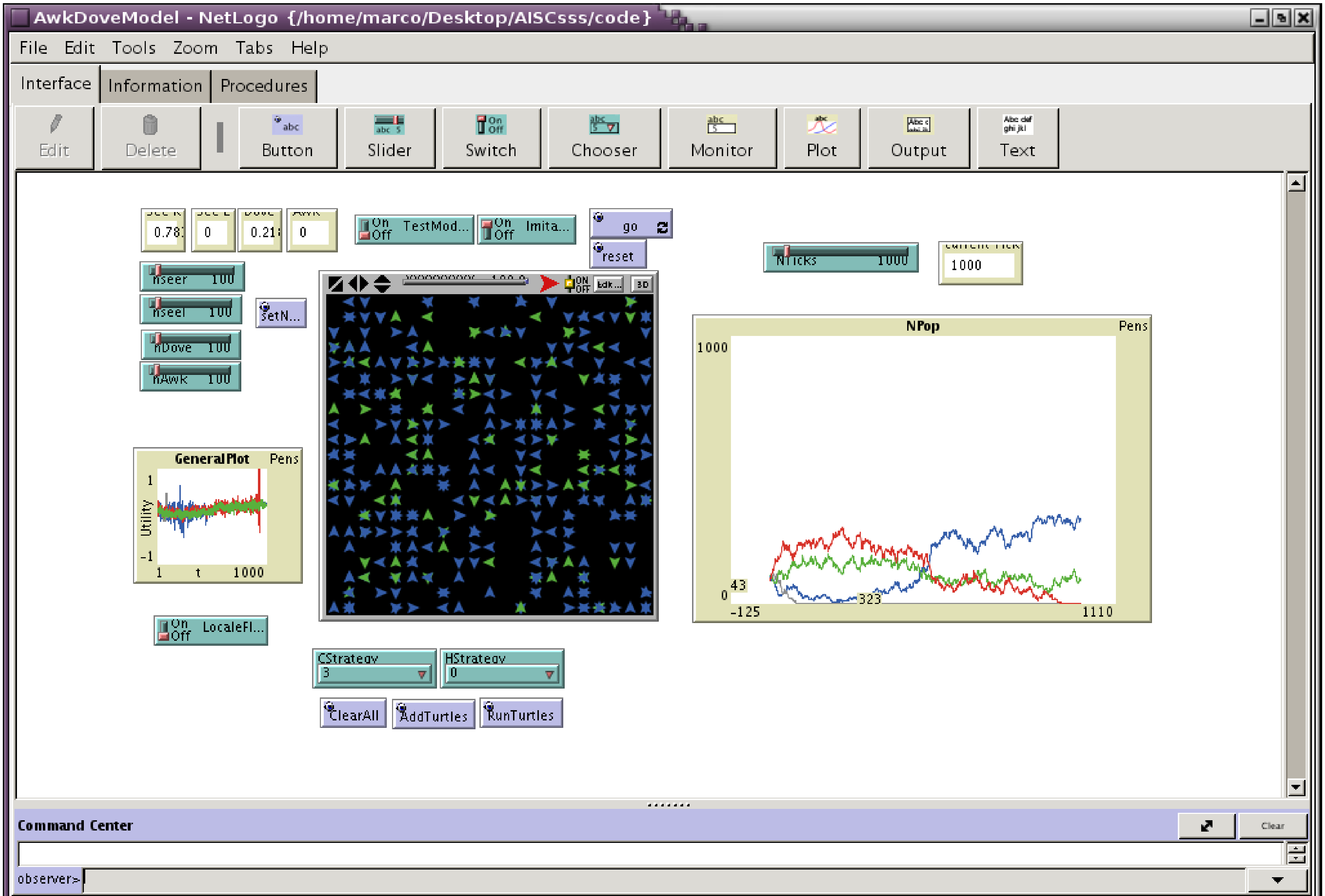
- We call the behaviors *i*, $r = 1$, $l = 2$, $d = 3$, $h = 4$.
- The general rules are:
  - if agent STOPS, the payoff is set to *0*.
  - If agents CRASH their payoffs are set to *-1*.
  - If an agent GOESAHEAD, its payoff is 1.

$$\Pr(\boldsymbol{h}|x_1, x_2, x_3, x_4) = \frac{\binom{x_1}{h_1}\binom{x_2}{h_2}\binom{x_3}{h_3}\binom{x_4}{h_4}}{\binom{N}{k}}$$

- We call a_k the number of cells with *k* individuals.
- We call K_h^i the matrix of payoffs different from zero for behavior *i* and combination *h*.
- The probability of combination h with $h1 + h2 + h3 + h4 = k$ conditioned to frequencies is a multinomial distribution,
- we can define the payoff for strategy *i*

$$\pi^i(x) = \sum_{k=2}^{4} a_k \left[\sum_{\boldsymbol{h}|k} \Pr(\boldsymbol{h}|x) K_{\boldsymbol{h}}^i \right]$$

The NetLogo Model



Simulations' Results 1/3

- We run 256 simulations varying the size of each sub-population of agents between 50 and 200, (in NetLogo [50 50 200])
- so the population's range varies from 200 to 800 agents. Each simulation includes 1000 ticks.
- (1) one sub-population survives:
  - (a) Right-watchers: 3 times (1.18%);
  - (b) Left-watchers: 5 times (1.95%);
- (2) two sub-populations survive:
  - (a) hawks and doves: 48 times (18.75%);
  - (b) right-watchers and doves: 15 times (5.86%);
  - (c) left-watchers and doves: 19 times (7.42%);
- (3) three sub-populations survive:
  - (a) right-watchers and hawks and doves: 85 times (33.20%);
  - (b) left-watchers and hawks and doves: 81 times (31.64%);

Simulations' Results 2/3

- *One sub-population survives*: in this situation the sub-population can only be either right watching or left watching, suggesting that, to put it in Lewis's terms, these are self-sufficient alternative solutions to problems of coordination. Being the two solutions equivalent, the choice between them is purely arbitrary. Thereby, once one solution has been selected, the other one cannot coexist.
- *Two sub-populations survive*. In this case,
  - as it happens with the Evolutionarily stable strategy (ESS) (see (Smith 1974; Gilbert 1981)), if one sub-population is hawks, the other one will necessarily be doves; hawks do not survive with either right or left watching sub-populations;
  - if, on the contrary, one of the sub-populations is doves, the other one may be any of the three remaining subpopulations, i.e. right watchers, left watchers or hawks.
  - Indeed, unconditioned strategies are not symmetrical: the former can survive only by exploiting the most altruistic strategy.
- *Three sub-populations survive* by including either right watchers or left watchers sub-populations, but never both at once.
- *Four sub-populations cannot survive* (this is a consequence of the previous item).

Simulations' Results 3/3

- Summing all of the percentages, we can calculate all of the cases in which each sub-population (alone or with others) survives. We can propose a hierarchy of adaptability:
  - (1) doves: 96.87%;
  - (2) hawks: 83.59%;
  - (3) right-watchers: 40.24%;
  - (4) left-watchers: 41.01%.

1) **M. GILBERT, (1981) "Game theory and convention", *Synthese*, 46:41–93.**

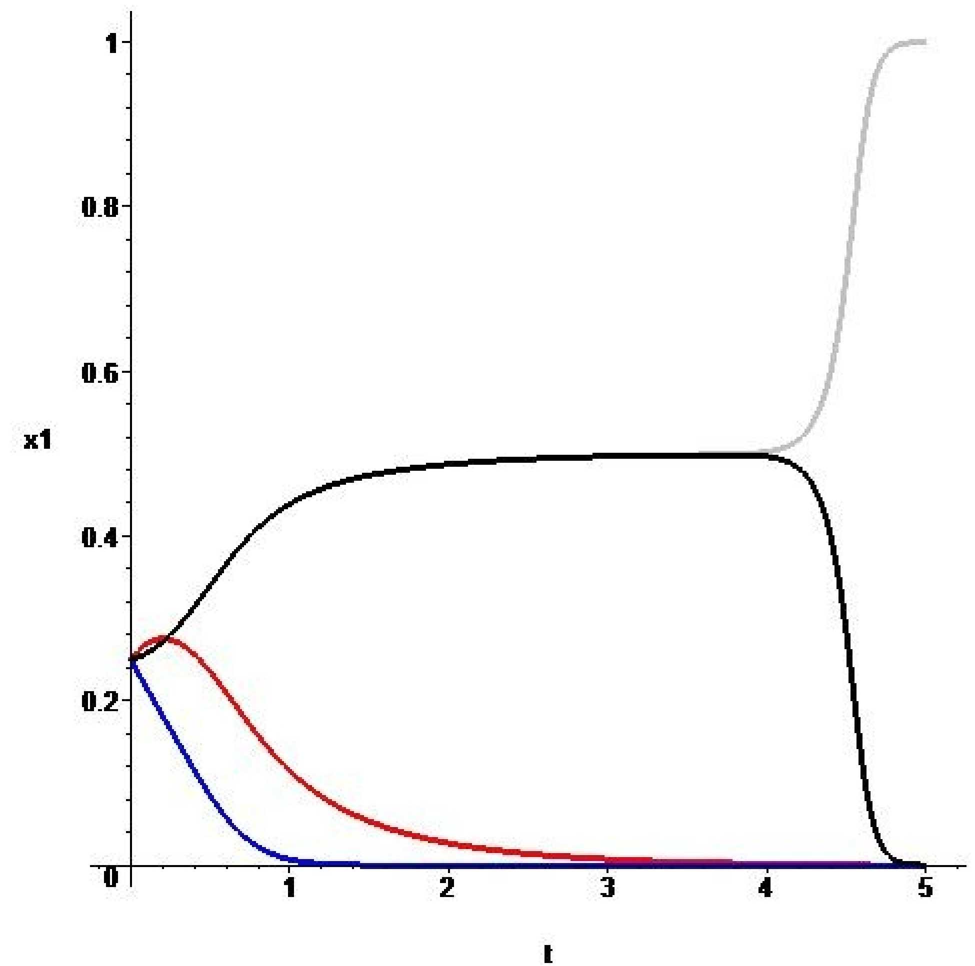
2) **J.M. SMITH. (1974) "The theory of games and the evolution of animal conflicts", *Journal of Theoretical Biology*, 47:209–21.**

Analytic Results

- We can extract information from the analytic model in three different ways.
  - (1) we can numerically solve the differential equations' system;
  - (2) we can draw the vectorial field described by the ODE (Open Dynamics Engine) system;
  - (3) finally, we can compute the steady states and the gradients around them, to valuate if the steady states are stable or not.

(1) we can numerically solve the differential equations' system

- A numerical solution of the analytic model. The red curve shows the frequency of Hawks, the blue shows the frequency of Doves. The gray and the black curves show the frequencies of conventional strategies. The initial state is almost uniform. We show that, after a delay, a conventional strategy wins (in this case the steady state is one-subpopulation).

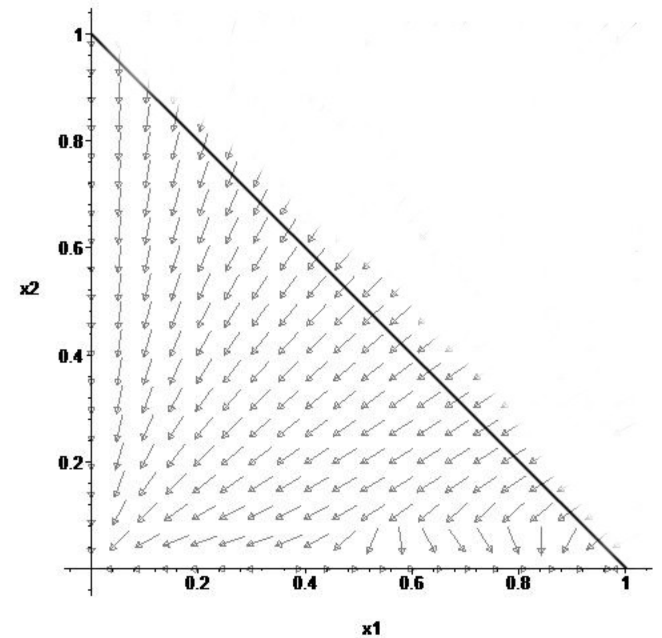


- The dynamic show that the ***conventional*** strategies are ***incompatible***.

(2) we can draw the vectorial field described by the ODE system

- We show the direction of the vector field for the *x1=right-watchers* and *x2=left-watchers*.
- In the figure 1, we set the number of hawks to zero.
- In the figure 2, we set the number of doves to zero.
- We see that with no hawks (1), the steady state with two sub-populations is feasible.

1



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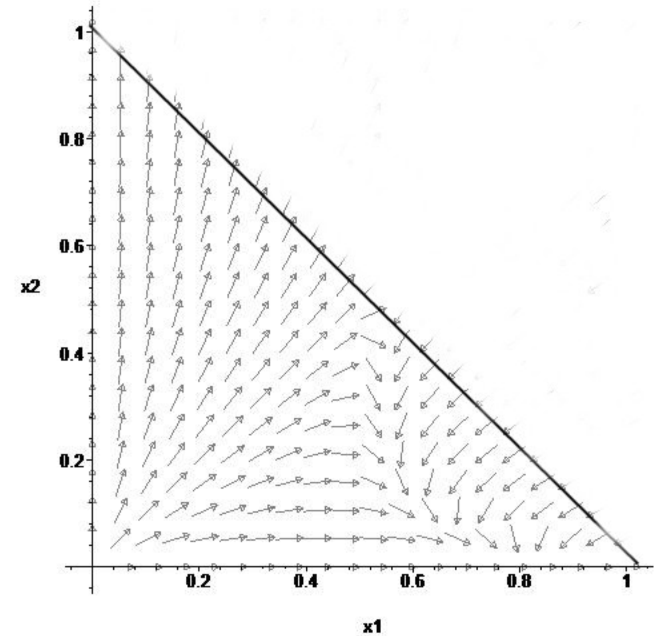
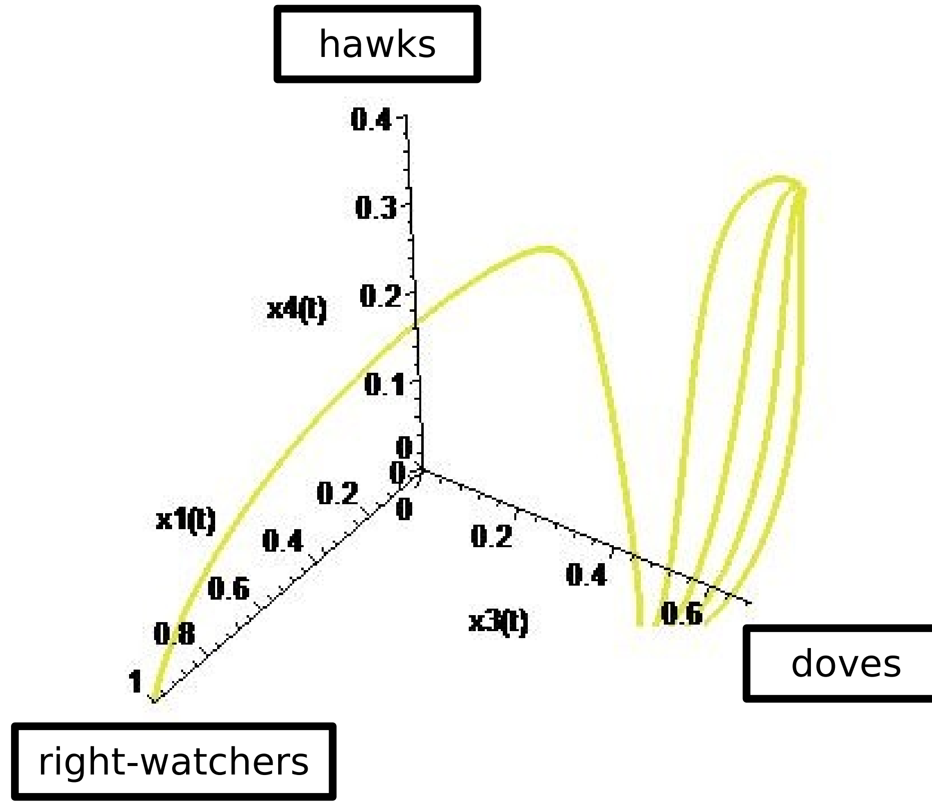


(3) finally, we can compute the steady states and the gradients around them, to valuate if the steady states are stable or not

- We have a map from the initial states with different numerousness to the steady states.
- In the figure we show that, starting with an average number of doves and a medium N, we have four path on five to two-subpopulation steady states.
- (*x1, x3, x4* means: *right-watchers, doves* and *hawks.* We set *left-watchers* to zero*)*



Discussion

- The Conditioned strategies WatchRight and WatchLeft are incompatible (see simulations' results and analytic result (1))
- Analytic result (1) shows that we have a delay before *separation* between the Condioned strategies (the delay is observed during the simulation too). Thus, we could argue that the delay is not a stochastic effect coming from the fluctuations, during the agent-based simulation. The delay is the effect of the *non-linearity* of the interactions
- The Conditioned strategies on one hand and the Aggressive strategy on the other are incompatible. Vice versa, we find a stable steady state with Conditioned strategies combined with the Compliant strategy (see analytic result (2))
- By the EBM (Equation-Based Model), we have a general map (see analytic result (3)) of the paths from the initial states, with different frequencies, to final steady states. The map shows a correspondence with the steady states' distribution from the simulations

Thank you very much for your attention...